

Who will relegate? A statistical analysis of the relegation struggle in the German Bundesliga 2015/16 based on bookmakers' odds

Gunther Schauburger & Andreas Groll
Department of Statistics, LMU Munich

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With only two matchdays left, the relegation struggle in the German Bundesliga has become highly dramatic. Still, 8 out of the 18 teams are in danger of being relegated to the 2. Bundesliga. The current situation of the regarding teams can be seen in the following table:

	Position	Points	Goals	Diff
⋮	⋮	⋮	⋮	⋮
 WOB	10	39	43:48	-5
 HSV	11	38	37:44	-7
 AUG	12	37	40:48	-8
 HOF	13	37	38:49	-11
 DAR	14	35	36:50	-14
 BRE	15	34	49:65	-16
 FRA	16	33	33:51	-18
 STU	17	33	48:69	-21
 HAN	18	22	29:59	-30

Only Hannover 96 is already determined to end the league on the last place. The team on place 17 is relegated directly to the 2. Bundesliga, the team on place 16 has to play deciding matches against the third-placed team from the 2. Bundesliga.

The aim of the following analysis is to estimate the chances of the remaining teams to end the season on positions 17 or 16. On the last two matchdays, in 12 out of the 18 matches teams are involved which are in danger to be relegated. The online bookmaker *Unibet* provides odds for all matches of the remaining two matchdays which can be used to calculate probabilities (after being adjusted for the bookmakers' margins) for each of the possible match outcomes (1: home team wins, X: draw, 2: away team wins). The respective probabilities in % can be seen in the following table:

				1	X	2
 FRA	-	DOR		11.19	19.41	69.41
 HSV	-	WOB		36.02	27.27	36.71
 HAN	-	HOF		26.8	24.4	48.8
 BER	-	DAR		55.93	26.78	17.29
 KOE	-	BRE		32.28	25.39	42.32
 S04	-	AUG		57.72	23.23	19.05
 STU	-	MAI		44.97	24.76	30.27
 AUG	-	HSV		45.27	27.96	26.78
 DAR	-	MGB		29.73	27.98	42.29
 HOF	-	S04		32.84	25.74	41.41
 BRE	-	FRA		42.38	26.86	30.76
 WOB	-	STU		43.24	26.07	30.69

Methodology and results

If per match only these three outcomes are distinguished, there are in total $3^{12} = 531\,441$ possible ways how the last two matchdays could end. Following the probabilities of the table above, for each of these outcomes a specific probability can be calculated. After all, in many cases, the final table can only be determined if also the specific match results (numbers of goals per team) are considered because many scenarios lead to teams with equal numbers of points. In this case, the team with a better (higher) goal difference is ranked higher. In the case of equal goal differences, the team which scored more goals is ranked higher. An exact analysis that takes into account all possible final goal differences would become dramatically more complex. If, for example, one would want to regard all possible outcomes distinguishing between the 10 most frequent results in football, already 10^{12} possible final outcomes would have to be distinguished.

However, we will use an analysis of the previously mentioned $3^{12} = 531\,441$ possible scenarios which still provides a highly accurate estimation of the chances of the single teams. The main assumption of the analysis is that the bookmakers' odds provide realistic assessments of the true probabilities. But, in order to nevertheless include precise match results, in a second step each single three-way outcome is allocated with an exact result. Based on the relative frequencies of single results known from the previous season, per scenario final results are randomly drawn. Therefore, rather frequent results like 1-0, 2-0 or 0-0 are more likely to be drawn than rare results like 0-4. Still, per match the probabilities from the table above are used to weight the probabilities of the single scenarios. This method allows to build a final table per scenario (531 441 tables) with an unambiguous ranking as per scenario the number of points and all numbers of goals are known per team.

Finally, this allows us to provide probabilities per team for the possible final standing of the Bundesliga which is shown in the following table:

		Place 17	Place 16	Staying up
	WOB	0.0000 %	0.0005 %	99.9995 %
	HSV	0.0000 %	0.0935 %	99.9065 %
	AUG	0.0012 %	0.4659 %	99.5329 %
	HOF	0.0071 %	0.5558 %	99.4371 %
	DAR	4.49 %	21.71 %	73.81 %
	BRE	12.17 %	15.88 %	71.95 %
	FRA	51.27 %	26.71 %	22.02 %
	STU	32.06 %	34.59 %	33.35 %

There is only a tiny chance of VfL Wolfsburg being on place 16 in the final table because of their current number of points and their good goal difference. Also for Hamburger SV, FC Augsburg and TSG Hoffenheim there is only a small chance left to be relegated, Hoffenheim has a chance of 0,56% to be on place 16 (which would make them play deciding matches).

All the other teams are still highly endangered, also SV Darmstadt still has a probability of 21,7% of playing deciding matches. Although currently not being on place 17, Eintracht Frankfurt has by far the highest risk of ending the league on place 17. In combination with their high probability of ending the league on place 16, Eintracht Frankfurt is the most likely candidate to follow Hannover 96 into the second division. For VfB Stuttgart, although currently placed on 17, the probabilities of all three possible outcomes are more or less equally likely.

In sum, still 31 different constellations for the last two positions are theoretically possible. The following table presents the 10 most likely constellations, together with their respective probabilities.

Place 16	STU	FRA	DAR	BRE	BRE	STU	FRA	DAR	DAR	STU
Place 17	FRA	STU	FRA	FRA	STU	BRE	BRE	STU	BRE	DAR
%	27.6	21.1	15.1	7.6	7.29	5.25	3.89	3.63	2.94	1.77

With a probability of 27.6%, the most likely final standing is that Frankfurt and Stuttgart switch places.

The all-or-nothing game

Clearly, the most important match for the relegation struggle among all remaining matches is the match between Werder Bremen and Eintracht Frankfurt on the last match day. Both teams are still highly in danger of being relegated. In total, Werder Bremen has better chances, mostly due to the fact that they are the home team in the respective match against Frankfurt. However, the result of this match is also very decisive for all the other teams. Therefore, we repeated the analyses from above, separately for the three possible outcomes of this match, i.e. for the scenarios "Werder Bremen wins", "Eintracht Frankfurt wins" or "Draw". Of course, all the following probabilities are only hypothetical as they include probabilities for matches previous to the match of interest.

- Werder Bremen wins

		Place 17	Place 16	Staying up
	WOB	0.0000 %	0.0002 %	99.9998 %
	HSV	0.0000 %	0.1590 %	99.8410 %
	AUG	0.0000 %	0.7485 %	99.2515 %
	HOF	0.0000 %	0.8300 %	99.1700 %
	DAR	1.77 %	25.28 %	72.95 %
	BRE	0.00 %	2.99 %	97.01 %
	FRA	77.64 %	20.44 %	1.92 %
	STU	20.59 %	49.56 %	29.85 %

- Eintracht Frankfurt wins

		Place 17	Place 16	Staying up
	WOB	0.0000 %	0.0002 %	99.9998 %
	HSV	0.0000 %	0.0359 %	99.9641 %
	AUG	0.0030 %	0.1770 %	99.8199 %
	HOF	0.0224 %	0.3597 %	99.6179 %
	DAR	9.91 %	19.19 %	70.91 %
	BRE	36.61 %	26.33 %	37.06 %
	FRA	8.27 %	30.87 %	60.85 %
	STU	45.18 %	23.04 %	31.78 %

- Draw

		Place 17	Place 16	Staying up
	WOB	0.0000 %	0.0000 %	100.0000 %
	HSV	0.0000 %	0.0111 %	99.9889 %
	AUG	0.0011 %	0.3522 %	99.6467 %
	HOF	0.0092 %	0.4251 %	99.5657 %
	DAR	2.81 %	18.84 %	78.35 %
	BRE	3.34 %	24.22 %	72.44 %
	FRA	58.82 %	32.07 %	9.11 %
	STU	35.02 %	24.08 %	40.90 %

Of course, this match has huge consequences for the chances of both teams. It can be seen that Werder Bremen is mostly in control of its own destiny. No matter how the next match of Werder Bremen against 1. FC Köln ends, Werder will not be on 17th place if they win the last match against Eintracht Frankfurt. Frankfurt would end the league on place 17 with a probability of 77,6%. In contrast, if Frankfurt wins the probability to end on place 17 is only 8,3%. However, the match also has consequences for the other teams. Both Darmstadt and Stuttgart would profit of a draw between Bremen and Frankfurt while a victory of Frankfurt enlarges their risk.

Whatever happens, now you know its probability!